ASYMPTOTIC STATE VECTOR COLLAPSE AND QED NONEQUIVALENT REPRESENTATIONS

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Abstract

The state vector evolution in the interaction of measured pure state with the collective quantum system or the field is analyzed in a nonperturbative QED formalism. As the model example the measurement of the electron final state scattered on nucleus or neutrino is considered. The produced electromagnetic bremsstrahlung contains the unrestricted number of soft photons resulting in the total radiation flux becoming the classical observable, which means the state vector collapse. The evolution from the initial to the final system state is nonunitary and formally irreversible in the limit of the infinite time.

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1 Introduction

The problem of the state vector collapse description in Quantum Mechanics (QM) is still open despite the multitude of the proposed models and hypothesis (D'Espagnat, 1990). This paper analyses some microscopic dynamical models of the collapse - i.e. the models which attempt to describe the interaction and the joint evolution of the measured state (particle) and the measuring device D (detector) from the first QM principles. Currently the most popular of them are the different variants of Decoherence Model which take into account also the interaction of the environment E with very large number degrees of freedom (NDF) and D with small NDF (Zurek,1982). Yet this model meets the serious conceptual difficulties resumed in so called Environment Observables Paradox (EOP) (D'Espagnat,1990). It demonstrates that for any decoherence process at any time moment at least one observable $\hat{B}$ exists which expectation value coincides with the value for the pure state and differ largely from the predicted for the collapsed mixed state. Moreover it follows that in principle it's possible to restore the system initial state which contradicts with the irreversibility expected for the collapse. In general EOP can be regarded as an important criteria of the measurement models correctness.

Meanwhile it was proposed that due to the very large internal NDF of the real macroscopic detectors the problem can be resolved by the methods of nonperturbative Quantum Field Theory (QFT) , which study the dynamics of the systems with the infinite NDF (Neeman,1985),(Fukuda,1987). The states manifold of such systems is described by Unitarily Nonequivalent (UN) representations, which permit to resolve EOP as will be demonstrated below.

The main difficulty of this approach is that it can be correctly applied only for the measurements on the systems with not simply very large, but exactly infinite NDF. Meanwhile the practical measuring devices must have the finite mass and energy. Here we consider QED bremsstrahlung model of the collapse which satisfy to all this demands simultaneously and without contradictions. It evidences also that the collapse-like processes can occur not only in the macroscopic objects, but also on the fundamental level of the elementary particles and fields.

Nonperturbative methods in QED were applied successfully for the study of the photon bremsstrahlung produced in any processes of the charged particles scattering on some target. The total number of produced photons with the energy larger than k_0, is proportional to $e^2 ln\frac{P_e}{k_0}$,i.e. it grows unrestrictedly as $k_0 \to 0$.. The perturbative Feynman diagram methods by definition works only for the processes which probability is small , while in this case it approximates to 1 (Itzykson,1980). The nonperturbative formalism was developed initially for the semiclassical case when the charges movement is prescribed (classical) and the back-reaction of the radiated electromagnetic (e-m) field $\hat{A}_\mu(x)$ on the charge movement can be neglected - BRF condition (Friedrichs,1953). Consequently in this case electromagnetic current $J_\mu(x)$ is not the operator ,but c-value and for the single electron scattering its 4-dimensional Fourier transform is equal to:

$$J_\mu(k) = ie(\frac{p_\mu}{pk} - \frac{p'_\mu}{p'k}) \qquad (1)$$

where p, p' are the initial and final e 4-momentum. In this case BRF condition means that the sum of radiated photons momentum $|\vec{k}_s|$ is much less then the electron momentum transfer in the scattering $|\vec{p} - \vec{p}'|$ (Akhiezer,1981).

Final e-m filed state is found by the nonperturbative calculation of S-matrix (S-operator) - T-product of interaction Hamiltonian density $\hat{H}_i(x) = \hat{H}_{em}(x) = \hat{J}_\mu(x)\hat{A}_\mu(x)$. $\hat{A}_\mu(x)$ is taken in Feynman gauge with the indefinite metric. The commutator of $\hat{H}_i(x), \hat{H}_i(x')$ is c-value function for the regarded c-value currents J_μ, which permits to transform T-product into the product of the integrals over 4-space :

$$\hat{S}_{em}(J) = exp[i\phi(J) - i\int \hat{H}_i(x)d^4x] = exp[(i-1)V(J) + U(J)] \tag{2}$$

where

$$U(J) = i\sum_{\lambda=1,2}\int d\tilde{k}[J_\mu(\vec{k})e^\lambda_\mu a^+(\lambda,\vec{k}] - J^*_\mu(\vec{k})e^\lambda_\mu a(\lambda,\vec{k})), \tag{3}$$

$$V(J) = \frac{1}{2(2\pi)^3}\int d\tilde{k}J^*_\mu(\vec{k})J_\mu(\vec{k})$$

where $d\tilde{k} = \frac{d^3k}{k_0}$,$a(\lambda,\vec{k})$ is the photon annihilation operator (Friedrichs,1953). Below we'll omit the sum over the photon polarization indexes λ or the polarization vectors e^λ_μ, where it's unimportant. $\phi(J) = V(J)$ is equal to the quantum phase between in- and out- states, if the relation $J^*_\mu(k) = J_\mu(-k)$ is fulfilled, which is true for J_μ of (1). As easy to see from (3) the amplitudes of the production of the photons with the different momentum $\vec{k}$ are independent. If the initial e-m field state is vacuum $|\gamma_0\rangle = |0\rangle$, then the average number of the produced photons is $d\bar{N}_{\vec{k}} = c|J_\mu(\vec{k})|^2 d\tilde{k}$. The action of $\hat{S}_{em}(J)$ results in the divergent photon spectra $d\bar{N}_\gamma = c\frac{dk_o}{k_0}$, for $J_\mu(k)$ of (1). It means that the final asymptotic state $|f\rangle$ includes the infinite number of very soft photons which total energy is finite (Jauch,1954). In the same time it gives :

$$|\langle f|0\rangle| = exp[-V(J)] = exp(-\frac{\bar{N}_\gamma}{2})$$

It follows that the state $|f\rangle$ doesn't belong to initial photon Fock space H_F, but to the different Hilbert space orthogonal to H_F. So the complete field states manifold M_c becomes nonseparable, i.e. described by the tensor product of the infinitely many Hilbert spaces H_i, each of them having its own cyclic vector - vacuum state $|0\rangle_i$. Any state of M_c is defined by two indexes $|\psi_j\rangle_i$, $i = 0$ corresponds to H_F. Remind that any Hermitian operator $\hat{B}$ - observable transforms only vectors inside the same Hilbert space $|\psi_2\rangle_i = \hat{B}|\psi_1\rangle_i$, and due to it for the arbitrary $|\psi_1\rangle_i, |\psi_2\rangle_l$, $i \neq l$, $\langle_i\psi_1|\hat{B}|\psi_2\rangle_l = 0$. So if the final state is the superposition of the states from different spaces $|f\rangle = |f_1\rangle_i + |f_2\rangle_l$ the interference terms (IT) for any $\hat{B}$ between $|f_1\rangle_i, |f_2\rangle_l$ are equal to zero. Consequently any measurement performed on such disjoint states can't distinguish the mixed and the pure initial states, which permit to resolve mentioned EOP for UN representations. Note that the bremsstrahlung due to the charge classical motion results in the final e-m field state which can belong only to the single Hilbert space H_i. Consequently to obtain the final disjoint states described QED formalism must be extended to incorporate the bremsstrahlung of the charged particles states superpositions, which will be done in this paper.

The transition from H_F to some H_i corresponds to Bogolubov boson transformation of the free field operators $a(\lambda.\vec{k}), a^+(\lambda, \vec{k})$

$$b_(\lambda, \vec{k}) = a(\lambda, \vec{k}) + iJ_\mu(\vec{k})e^\lambda_\mu \quad (4)$$

which is nonunitary for $J_\mu(\vec{k})$ of (1), but conserves the vector norm $\langle f|f\rangle = \langle 0|0\rangle$.

2 QED Measurement Model

As the example of the collapse induced by the bremsstrahlung we'll regard Weak scattering of the electron on the neutral particle (neutrino) ν with mass m_0, which in principle can be zero. We can consider also the electron coulomb scattering on the nucleus ,but its infinite range results in the infinite total cross section which introduce the unnecessary complications. We'll show that the final photon bremsstrahlung disjoint states formally measure whether the act of scattering took place or the particles passed unscattered and conserved their initial state. In the same time it's the measurement of the e helicity λ_e, because for its left, right helicities cross-sections $\sigma_L >> \sigma_R$ in weak interactions.

Now e motion is nonclassical and defined by e field operators the general S-operator for $\hat{H}_i(x) = \hat{H}_{em}(x) + \hat{H}_w(x)$ should be found. Here we'll describe the method of its matrix elements $\langle f|\hat{S}|i\rangle$ calculations for the states of interest without finding S-operator analytical form, which is quite difficult. This nonperturbative calculations are possible for the soft photon radiation for which BRF condition is fulfilled, i.e. the total e-m field recoil is much less then e momentum transferred to ν, as was discussed in chap.1. It means that $\hat{H}_{em}(x)$ doesn't act on e field operators conserving its spin and momentum and acts only on e-m field operators (Jauch,1954). On the contrary $\hat{H}_w$ acts only on e, ν fields, and due to it it's possible to factorise S-operator into $\hat{S}_w$ and $\hat{S}_{em}$ parts. $\hat{S}_w$ defines the skeleton diagram which defines solely the final e', ν' states, which is dressed by the soft radiation given by $\hat{S}_{em}$. In its turn $\hat{S}_{em}$ and consequently the final radiation field depends on final e momentum, defined by $\hat{S_w}$ action on the initial state.

So we should start from the calculation of $\hat{S}_w$ action on the initial e, ν states neglecting $\hat{H}_{em}(x)$. The smallness of the weak interaction constant G permits to calculate $\hat{S_w}$ perturbatevely with the good accuracy, and at c.m.s. energies below 1 TeV , which we'll regard here, its calculation can be restricted to 1st order diagram (Cheng,1984). Its amplitude M_w for the weak vertex $e, \nu \to e', \nu'$ results in the spherically symmetric distribution of e',ν' in the c.m.s :

$$M_w(e', \nu') = \langle e, \nu|\hat{S}^1_w|e', \nu'\rangle = \frac{G}{\sqrt{2}} J_{L\mu} J^*_{L\mu} = \bar{u'}_e \gamma_\mu(1 + \gamma_5)u_e \bar{u'_\nu}\gamma_\mu(1 + \gamma_5)u_\nu. \quad (5)$$

From it we can find the final e-m field state if we know the operator $\hat{S}_{em}(J^l)$ for the initial and final momentum eigenstates $|e\rangle, |e'_l\rangle$. Despite that now e electromagnetic current is formally the operator, it was found that $\hat{S}_{em}(J^l)$ is independent of the initial and final e polarisations and described by the formulae (2) in which current

Fourier transform is equal to $J^l_\mu = J_\mu(k, p, p'_l)$ of (1), where p, p'_l are the corresponding eigenvalues (Jauch,1954). This result doesn't seems surprising , because such states describe the prescribed e motion in the phase space. Then, as follows from the superposition principle, if e final momentum eigenstate $|e'_l\rangle$ have the amplitude c_l ,the final system state is :

$$|\psi_f\rangle = \sum c_l |e'_l\rangle |\nu'_l\rangle \hat{S}_{em}(J^l)|\gamma_0\rangle$$

In our case it results in the final nonclassical system state which is the entangled product of e',ν' states and disjoint e-m field states

$$|f_w\rangle = \sum_{l=0} c_l |f\rangle_l = |f\rangle_\alpha + |f\rangle_0 = \sum_{l=1} M_w(e'_l, \nu'_l)|e'_l\rangle|\nu'_l\rangle|\gamma^f\rangle_l + M_0|e\rangle|\nu\rangle|0\rangle \qquad (6)$$

where $|\gamma^f\rangle_l = \hat{S}_{em}(J^l)|0\rangle$, the sum over l means the integral over the correlated final e', ν' momentum $p'_l, p'_{l\nu}$. M_0 is the zero angle amplitude of particles nonscattering. All the partial phases ϕ_l are infinite , moreover , as follows from (3) their differences δ_{lm} are divergent, as must be for the disjoint states :

$$\delta_{lm} = \int \frac{d\tilde{k}[J^{l*}_\mu(\vec{k})J^l_\mu(\vec{k}) - J^{m*}_\mu(\vec{k})J^m_\mu(\vec{k})]}{2(2\pi)^3} = F(\varphi_{lm}) \int \frac{dk_0}{k_o}$$

,where φ_{lm} is the angle between $\vec{p'}_l, \vec{p'}_m$. Due to it in the limit $t = \infty$ this process is formally completely irreversible ,because T-reflection of the sum of such states with the indefinite relative phases produces the new state completely different from the initial one.

Then ,as was stressed already, for such disjoint state any measurement of arbitrary Hermitan $\hat{B}$ will give : $\langle f_0|\hat{B}|f\rangle_\alpha = 0$. It means that we obtained in QED based model the final disjoint state with the finite total energy. Its components $|f\rangle_0$ and $|f\rangle_\alpha$ corresponds to the different values of e polarisation λ_e which we intended to measure. As the result this state have all the observable properties of the mixed state which must appear after this measurement. Note that it was obtained for the complete final state without averaging over some subsystem ,or regarding it as the unmeasurable environment (Zurek,1982). Formally this is the main result of our paper, yet it's important to discuss also the practical aspects of the continuous photons spectra measurements, and possible developments of QFT models for the real solid state detectors.

In practice $\hat{B}$ can be only the bounded operator in H_F ,because only this case corresponds to the photon measurements by the finite detectors ensemble (Itzykson,1980). Note that the practical direct IT observation is impossible even between the single photon $|\vec{k}\rangle$ and the vacuum states,as follows from Photocounting theory (Glauber,1963). It shows that all e-m field operators $\hat{B}_\gamma$ which can be measured are the functions of $\hat{n}(\lambda, \vec{k}) = a(\lambda, \vec{k})a^+(\lambda, \vec{k})$ - the photon number operators. But for such operators $\langle \vec{k}|\hat{B}_\gamma|0\rangle = 0$, and the same will be true for any states with the different photon numbers. To reveal IT presence for this single photon state the only possibility is to perform the special premeasurement procedure (PP), namely

$|\vec{k}\rangle$ must be reabsorbed by its source Q_γ and the interference of the source states for some new observable of the form $\hat{B}_s = a(\vec{k})\hat{B}$ studied. Yet, to our knowledge there is no general proof that such PP always exists for the multiphoton states with the continuous spectra. The famous Recurrence theorem is true only for the discrete spectra (Bocchiery,1957).

Such PP certainly doesn't exist for $|f_w\rangle$ states at $t = \infty$ due to discussed loss of relative phases between its parts $|f\rangle_l$. Really if the phase differences δ_{lm} are infinite for the sum of e-m field states ,then their reabsorbtion will mean that this loss of coherence is transferred to Q_γ state which after it will become disjoint. But we'll give the qualitative arguments that such PP probably doesn't exist also for this states taken at finite time.

As the example we'll regard PP layout in which the scattered e, ν are reflected by some very distant mirrors back to the interaction region where they can rescatter again. Then we must calculate e radiation appearing after three consequent collisions, taking into account also 'internal' e radiation between collisions. As Low theorem demonstrates e-m radiation field in the infrared limit in any process is defined solely by the current calculated between asymptotic in-,out- momentum eigenstates, neglecting intermediate states (Low,1958). It means that we can apply for the calculations the method described above and in particular the resulting formula (6).

Then the initial e-m field state restoration is defined by $\langle 0|\hat{S}_{em}(J^l)|0\rangle$ amplitude of $|0\rangle$ restoration in the e, ν rescattering ,which is nonzero only for $J_\mu(k) = 0$ as follows from (2). It means that e in- and out- momentum must coincide,and from the energy conservation the same be true for ν. So we must calculate the probability P of 2-nd order weak process $i \to v'_l \to v^r_l \to i$. Here v'_l are all possible intermediate states and v^r_l are reflections of v'_l in the nondispersive mirrors ,the reflection amplitude is supposed to be $M_r = exp(i\phi_c)$ and can be omitted. The calculation is simplified by the spherical symmetry of weak scattering and we obtain, omitting some unessential details :

$$P = \frac{\int |M_w(e'_l, \nu'_l) M_w(e^r_l, \nu^r_l \to e, \nu)|^2 do_v}{\int\int |M_w(e'_l, \nu'_l) M_w(e^r_l, \nu^r_l \to e_f, \nu_f)|^2 do_v do_f} = \frac{|\bar{M}_w|^4 o_v \delta^3(\vec{p_f} - \vec{p_e})}{|\bar{M'}_w|^4 o_v o_f} = 0$$

o_v, o_f are the phase spaces of intermediate and final e, ν states which are reduced to the corresponding e phase spaces. So o_v, o_f is isomorphic to the spherical surface with the radius $r = |\vec{p_e}|$ with the nearly constant density of the final states on it. $\bar{M}_w, \bar{M'}_w$ are the expectation values of M_w over the indicated phase volumes ,which are assumed to be of the same order. The restoration of the initial state corresponds to a single point $\vec{r_0} = \vec{p_e}$ on this surface. Each infinitely close point to $\vec{r_o}$ corresponds to another Hilbert space with the infinite number of soft photons. So the zero probability of the initial state restoration obtains the simple geometrical interpretation ; $\vec{r_0}$ is the single nonsingular point in the phase space which can be omitted without changing any physical result. Despite this arguments have qualitative character they demonstrate that the disjoint states evolution irreversibility is connected with the QM principal uncertainty of scattering angles.

It's important to note that such effect supposedly can exist also for the rescattering of the photon states with the finite NDF and continuous spectra which belong to H_F. If the proof of it will be given, on which we work now, then the conditions of the collapse observations in QED can become more tight and wouldn't demand the use of UN representations and disjoint states.

In QFT studies the situations in which the particular dynamics makes some operators unobservable are well known. The most famous example is QCD colour confinement where coloured charge is the analog of electric charge and QCD Hamiltonian contains infrared singularity induced by the massless bosons - gluons (Itzykson ,1980). Any attempt to measure coloured operators ,for example quark or gluon momentum, results in the soft gluon production. In a very short time this coloured quanta fuses into some number of colourless hadrons and consequently only the hadron operators are the real observables of this theory.

In practice the measurement is performed on the localized states (wave packets) and lasts only the finite time. Meanwhile it was shown that any localized charged state includes the unlimited number of soft photons (Buchholz,1991). It supposes that the structure of localized and nonlocalized states asymptotically coincides and their evolution will result in analogous disjoint final states.

The real detectors are the localized solid objects to which the regarded model can't be applied directly. Yet QFT methods were used very successfully in Solid State Physics , and so we can hope that they'll permit to describe the collapse in the real detectors. The example of such approach gives the simple model of the collapse induced by Ferromagnetic phase transition (Mayburov,1995).

It's well known that the solid state collective excitations - quasiparticles are massless and their excitation spectra have no gap i.e. infrared divergent (Umezawa,1982). This quanta readily interact with e-m field, so any excitation of the crystal in the vacuum is to be relaxated by the soft radiation. Its main mechanism is probably the cascade phonon decay $P \rightarrow P' + \gamma$. So the excitation of the crystal by the measured energetic particle can result in a new disjoint state of the crystal plus the external electromagnetic field. This idea can also be applicable for the finite system if its surface is regular and can be regarded as the topological defect with infinite NDF which results in a special kind of boson condensation in the crystal volume (Umezawa,1978).

In conclusion we've shown that final states of $e - \nu$ scattering asymptotically in the standard S-matrix limit reveal the properties of the mixed state i.e. perform the collapse. This doesn't seems a surprise ,because the classical features of electron bremsstrahlung states were stressed often (Kibble,1968). In addition this model can formally describes the radiation decoherence process of the special kind ,when the system being measured generates its environment from the initial vacuum.

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